https://asaf.pm/live

Oscilador armónico

$$\hat{N} = \hat{a}^{+}\hat{a} \qquad \hat{N}\,|\phi_n\rangle = n\,|\phi_n\rangle$$

$$\hat{a}^{+}|\phi_n\rangle = \sqrt{n+1}\,|\phi_{n+1}\rangle$$
$$\hat{a}\,|\phi_n\rangle = \sqrt{n}\,|\phi_{n-1}\rangle$$

Notación común

$$|\phi_n\rangle \equiv |n\rangle$$

$$|n\rangle = \frac{1}{\sqrt{n!}}(\hat{a}^{+})^{n}|0\rangle$$

Funciones de Onda

$$\langle x|\phi_n\rangle = \langle x|n\rangle = \phi_n(x)$$

proyectando con $\langle x|$

$$\hat{a}|\phi_0\rangle = 0 \quad \Longrightarrow \quad \langle x|\hat{a}|\phi_0\rangle = 0$$

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\underline{\hat{X}} + i\,\underline{\hat{P}}\right) = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{X} + i\frac{1}{\sqrt{m\omega\hbar}}\hat{P}\right)$$

$$\langle x|\frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{X} + i\frac{1}{\sqrt{m\omega\hbar}}\hat{P}\right)|\phi_0\rangle = 0$$

$$\frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\underbrace{\langle x|\hat{X}|\phi_0\rangle}_{x\,\phi_0(x)} + i\frac{1}{\sqrt{m\omega\hbar}}\underbrace{\langle x|\hat{P}|\phi_0\rangle}_{-i\hbar\frac{d}{dx}\phi_0(x)}\right) = 0$$

multiplicando por $\sqrt{2}\sqrt{\frac{m\omega}{\hbar}}$ de ambos lados.

$$\left(\frac{m\omega}{\hbar}x + \frac{d}{dx}\right)\phi_0(x) = 0$$

Solución: $$\phi_0(x) = C\,e^{-\frac{m\omega}{2\hbar}x^2}$$

Para normalizar

$$1 = \int_{-\infty}^{\infty} |\phi_0(x)|^2 dx = \int_{-\infty}^{\infty} |c|^2 e^{-\frac{m\omega}{\hbar}x^2} dx$$

$\sqrt{\frac{m\omega}{\hbar}}\, x \to s$

$\sqrt{\frac{m\omega}{\hbar}}\, dx = ds$

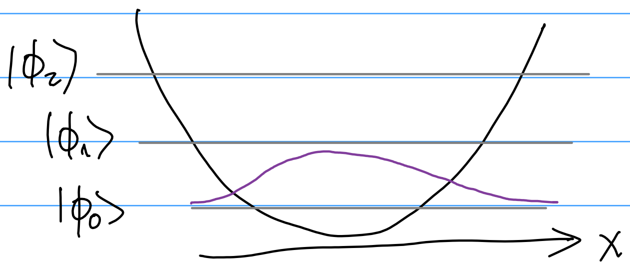
$$= |c|^2 \sqrt{\frac{\hbar}{m\omega}} \int_{-\infty}^{\infty} e^{-s^2} ds$$

$$= |c|^2 \sqrt{\frac{\hbar\pi}{m\omega}}$$

$$\Rightarrow \quad |c| = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4}$$

elegimos c real y positiva por comodidad

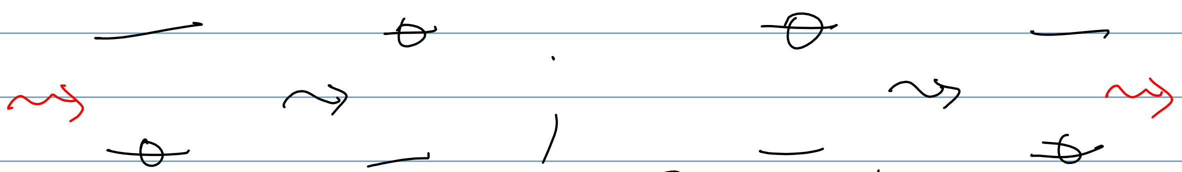
$$\therefore \quad \boxed{\phi_0(x) = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}}$$



$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right)$$

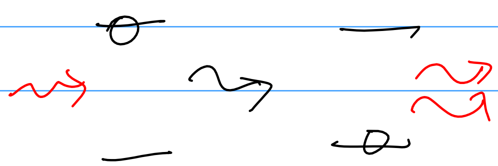
Distribución gaussiana $\quad N e^{-\frac{x^2}{2\sigma^2}}$

$$\sigma = \sqrt{\frac{\hbar}{m\omega}}$$



Absorción

Decaimiento espontáneo



Emisión estimulada

Para obtener $|\phi_1\rangle$, $\hat{a}^+|\phi_0\rangle = \sqrt{0+1}\,|\phi_1\rangle$

$$|\phi_1\rangle = \hat{a}^+|\phi_0\rangle \; ; \quad \underbrace{\langle x|\phi_1\rangle}_{\phi_1(x)} = \langle x|\hat{a}^+|\phi_0\rangle$$

$$\hat{a}^+ = \frac{1}{\sqrt{2}}\left(\hat{X} - i\hat{P}\right) = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{x} - i\,\frac{1}{\sqrt{\hbar m\omega}}\,\hat{P}\right)$$

$$\phi_1(x) = \frac{1}{\sqrt{2}}\left[\sqrt{\frac{m\omega}{\hbar}}\,\underbrace{\langle x|\hat{X}|\phi_0\rangle}_{x\,\phi_0(x)} \;\frac{-i}{\sqrt{\hbar m\omega}}\,\underbrace{\langle x|\hat{P}|\phi_0\rangle}_{-i\hbar\frac{d}{dx}\phi_0(x)}\right]$$

$$\phi_0(x) = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\phi_1(x) = \left[\frac{4}{\pi}\left(\frac{m\omega}{\hbar}\right)^3\right]^{1/4} x\, e^{-\frac{m\omega}{2\hbar}x^2}$$

Análogo

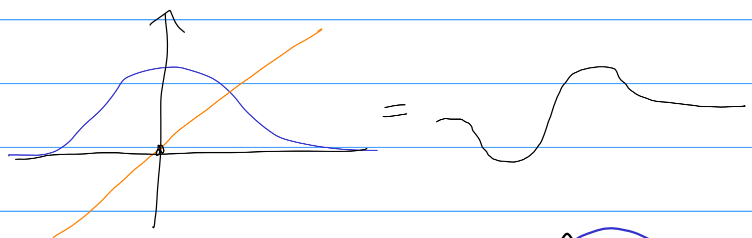
$$\phi_2(x) = \left(\frac{m\omega}{4\pi\hbar}\right)^{1/4}\left[2\frac{m\omega}{\hbar}x^2 - 1\right] e^{-\frac{m\omega}{2\hbar}x^2}$$

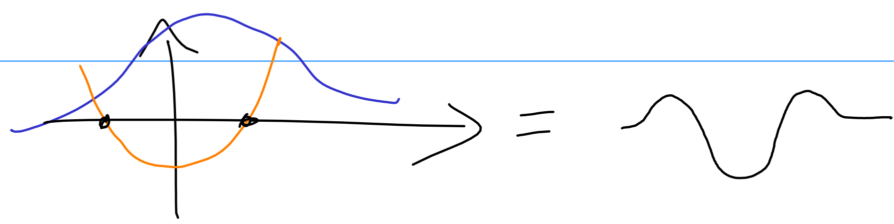
$$\phi_n(x) = \frac{1}{\sqrt{n!}}\langle x|(\hat{a}^+)^n|\phi_0\rangle$$

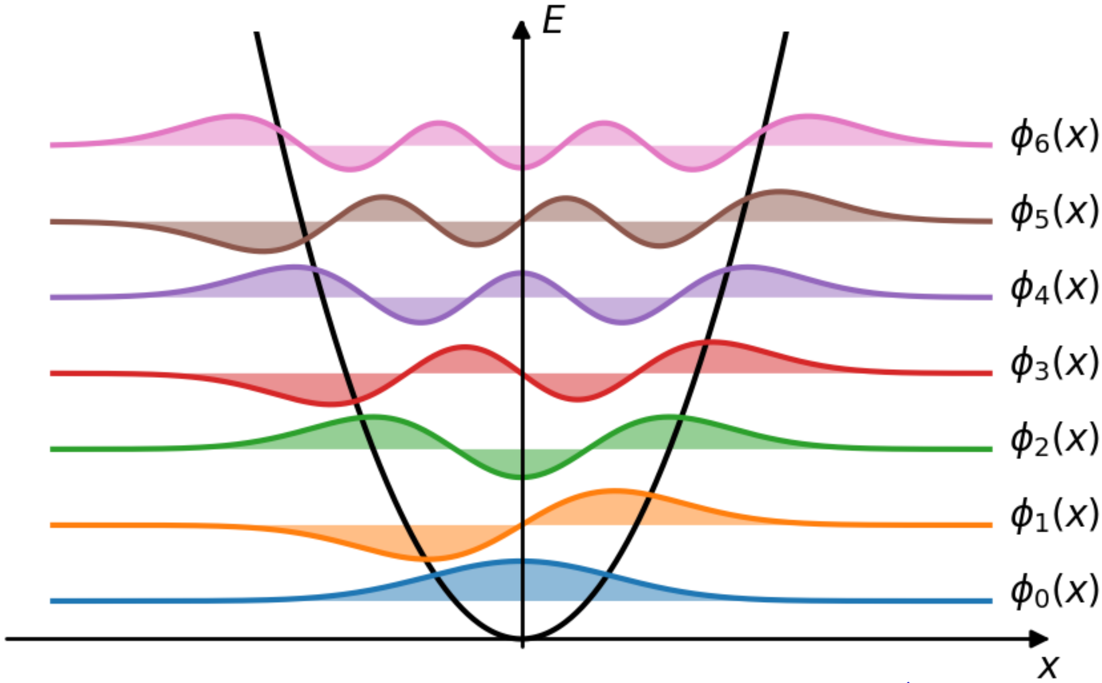
$$= \frac{1}{\sqrt{n!}}\frac{1}{\sqrt{2^n}}\left[\sqrt{\frac{m\omega}{\hbar}}\,x - \sqrt{\frac{\hbar}{m\omega}}\frac{d}{dx}\right]^n \phi_0(x)$$

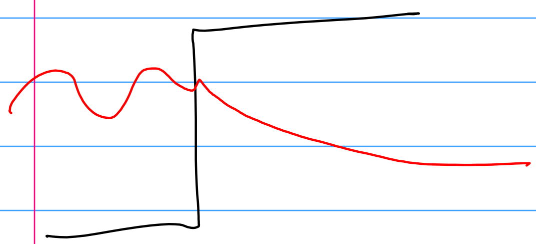
Quedan polinomios multiplicados por e^{-x^2}.

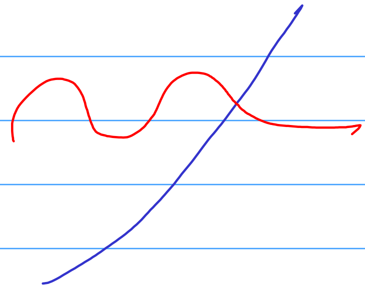
← polinomios de Hermite.









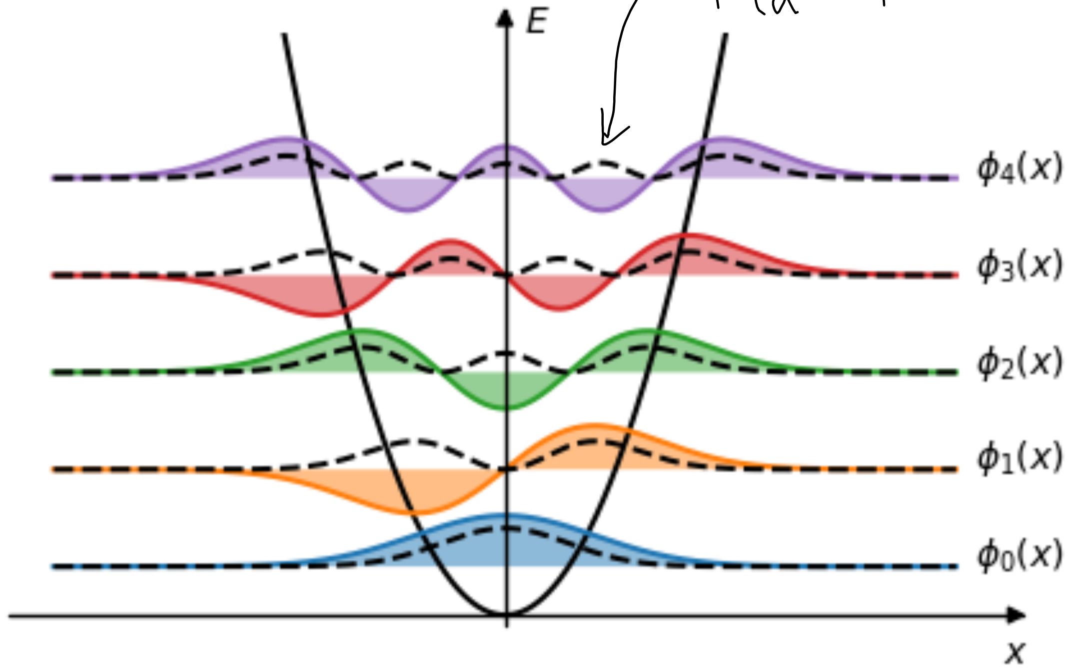


- al aumentar n es más ancha la distribución
  - tiene mayor energía potencial promedio

→ La n-ésima función tiene n nodos.

⇒ Más energía cinética promedio.

$|\phi_n(x)|^2$



Evolución temporal.

$$\hat{H}|\phi_n\rangle = E_n|\phi_n\rangle$$

$$|\psi(0)\rangle = \sum_n c_n |\phi_n\rangle$$

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_nt/\hbar} |\phi_n\rangle$$

Graficamos $e^{-iE_nt/\hbar}\phi_n(x)$