

- Podemos medirlos simultáneamente.
Es decir, medir alguno de ellos no borra la información del otro

Consideremos $[\hat{A}, \hat{B}] = 0 \Rightarrow$

$$\begin{aligned}\hat{A} |a_n, b_p, i\rangle &= a_n |a_n, b_p, i\rangle \\ \hat{B} |a_n, b_p, i\rangle &= b_p |a_n, b_p, i\rangle\end{aligned}$$

distinguir

¿Cuál es $P(a_n, b_p)$?
primero luego

$$P(A|B) = P(A), P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow P(A \cap B) = P(A|B)P(B) \rightarrow P(A \cap B) = P(A)P(B)$$

Consideremos $|\psi\rangle = \sum_{n,p,i} c_{n,p,i} |a_n, b_p, i\rangle$

Si medimos \hat{A}

$$P(a_n) = \sum_{p,i} |c_{n,p,i}|^2 \quad (C-32)$$

When we then measure B , the system is no longer in the state $|\psi\rangle$ but, if we have found a_n , in the state $|\psi'_n\rangle$:

$$|\psi'_n\rangle = \frac{1}{\sqrt{\sum_{p,i} |c_{n,p,i}|^2}} \sum_{p,i} c_{n,p,i} |a_n, b_p, i\rangle \quad (C-33)$$

The probability of obtaining b_p when it is known that the first measurement has yielded a_n is therefore equal to:

$$P_{a_n}(b_p) = \frac{1}{\sum_{p,i} |c_{n,p,i}|^2} \sum_i |c_{n,p,i}|^2 \quad (C-34)$$

The probability $P(a_n, b_p)$ sought corresponds to a "composite event": to be in a favorable case, we must first find a_n and then, having satisfied this first condition, find b_p . Therefore:

$$P(a_n, b_p) = P(a_n) \times P_{a_n}(b_p) \quad (C-35)$$

Substituting into this formula expressions (C-32) and (C-34), we obtain:

$$P(a_n, b_p) = \sum_i |c_{n,p,i}|^2 \quad (C-36)$$

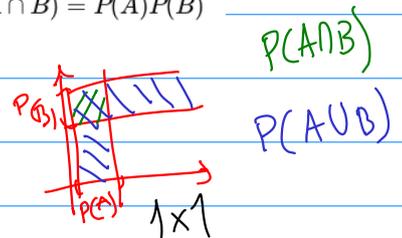
Moreover, the state of the system becomes, immediately after the second measurement:

$$|\psi''_{n,p}\rangle = \frac{1}{\sqrt{\sum_i |c_{n,p,i}|^2}} \sum_i c_{n,p,i} |a_n, b_p, i\rangle \quad (C-37)$$

Therefore, if we decide to measure either A or B again, we are sure of the result (a_n or b_p): $|\psi''_{n,p}\rangle$ is an eigenvector common to A and B with the eigenvalues a_n and b_p respectively.

Tarea:

$$P(a_n, b_p) = P(b_p, a_n) = \sum_i |c_{n,p,i}|^2 = \sum_i |\langle a_n, b_p, i | \psi \rangle|^2$$



$P(A|B) = P(A \cap B)$
 $\rightarrow P(B)$
para normalizar

El proyector como observable

Como $P_{|u_n\rangle} = P_{|u_n\rangle}^\dagger$ podemos tomarlo como observable.

Considerando una base ortonormal $\{|u_i\rangle : i \in I\}$

¿Cuáles son los e-valores y e-vectores de

$$P_{|u_n\rangle} = |u_n\rangle\langle u_n|$$

$$P_{|u_n\rangle} |u_n\rangle = |u_n\rangle$$

$$P_{|u_n\rangle} |u_i\rangle = |u_n\rangle\langle u_n | u_i\rangle = 0$$

\uparrow
 $i \neq n$

$$[P_{|u_n\rangle}]_{\{|u_i\rangle\}} = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & \ddots \\ & & & & & 0 \end{pmatrix}$$

$$\therefore \text{Para } |\psi\rangle = \sum c_n |u_n\rangle$$

$$\mathcal{P}(1) = |\langle u_n | \psi \rangle|^2 = |c_n|^2$$

El proyector $P_{|u_n\rangle}$ es un observable que mide si el sistema está o no en el estado $|u_n\rangle$.