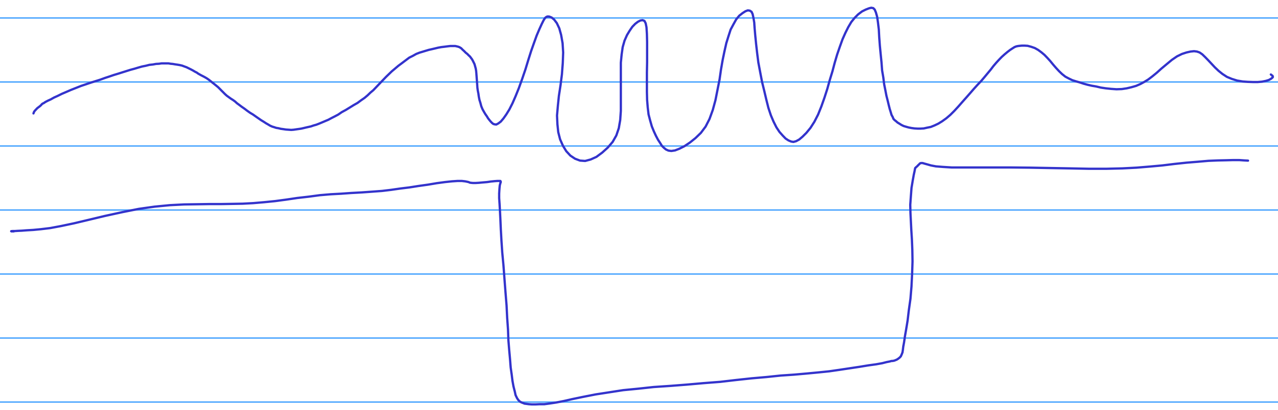
¿Qué pasa para E>0?

$\varphi_I(x) = A e^{ikx} + A' e^{-ikx}$
$\varphi_{II}(x) = A_{II} e^{ik_2x} + A'_{II} e^{-ik_2x}$
$\varphi_{III}(x) = A_{III} e^{ikx} + A'_{III} e^{-ikx}$

- No podemos determinar todo.
- no podemos normalizar.
- cualquier E>0 es válida

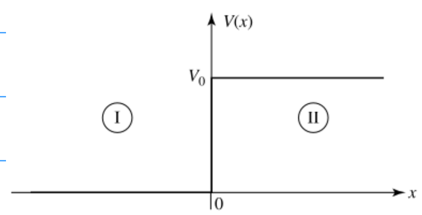


Esto significa que para E>0 debemos interpretar la función de onda distinto.

Para una condición inicial realista, tendremos un paquete de onda localizado que sí podemos normalizar (pero no es un estado estacionario).

Escalón

(es más fácil empezar aquí)



a) Caso E> V_0

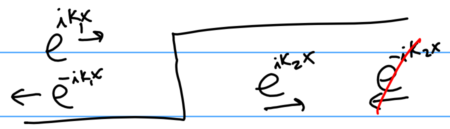
$$\sqrt{\frac{2mE}{\hbar^2}} = k_1 \qquad \sqrt{\frac{2m(E-V_0)}{\hbar^2}} = k_2$$

$$\varphi_I(x) = A_1 e^{ik_1x} + A_1' e^{-ik_1x}$$
$$\varphi_{II}(x) = A_2 e^{ik_2x} + A_2' e^{-ik_2x}$$

Con $\varphi_I(0) = \varphi_{II}(0)$ y $\varphi_I'(0) = \varphi_{II}'(0)$ no determinamos A_1, A_1', A_2, A_2'.

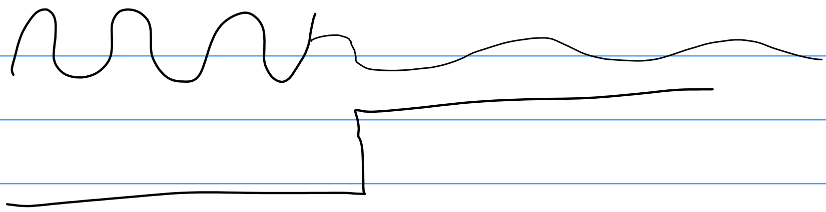
Aquí no aplica la normalización $\int |\varphi(x)|^2 dx = 1$

Por eso nos limitamos al caso $A_2' = 0$ que significa la partícula viene de la izq.



$$\vec{J} = \frac{\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right)$$

Para $\psi = A e^{ikx} + A' e^{-ikx}$ $\quad J = \frac{\hbar k}{m}\left[|A|^2 - |A'|^2\right]$



Es interesante $\quad R = \frac{J_{reflejada}}{J_{incidente}} = \left|\frac{A_1'}{A_1}\right|^2 \quad T = \frac{J_{trans}}{J_{incidente}} = \frac{k_2}{k_1}\left|\frac{A_2}{A_1}\right|^2$

$$\varphi_I(x) = A_1 e^{ik_1x} + A_1' e^{-ik_1x}$$
$$\varphi_{II}(x) = A_2 e^{ik_2x} + \cancel{A_2' e^{-ik_2x}}$$

$\varphi_I(0) = \varphi_{II}(0) \qquad A_1 + A_1' = A_2$

$\varphi_I'(0) = \varphi_{II}'(0) \qquad iA_1k_1 - iA_1'k_1 = iA_2k_2$

$$A_1k_1 - A_1'k_1 = (A_1 + A_1')k_2 \Rightarrow A_1(k_1 - k_2) = A_1'(k_1 + k_2)$$

$$\Rightarrow \frac{A_1'}{A_1} = \frac{k_1 - k_2}{k_1 + k_2}$$

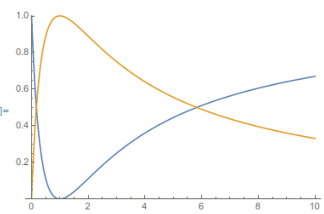
$$\frac{A_2}{A_1} = \frac{2k_1}{k_1 + k_2}$$

$$R = 1 - \frac{4k_2/k_1}{(1 + k_2/k_1)^2} \qquad T = \frac{4k_2/k_1}{(1 + k_2/k_1)^2}$$

Para $\frac{k_2}{k_1} = 1 \qquad R = 0, \; T = 1$

In[4]:= Plot[{1 - 4κ/(1+κ)^2, 4κ/(1+κ)^2}, {κ, 0, 10}, PlotRange → {0, 1}]

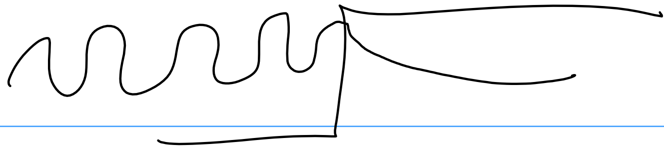
Out[4]=



In[5]:= Solve[1 - 4κ/(1+κ)^2 == 0, κ]

Out[5]= {{κ → 1}, {κ → 1}}

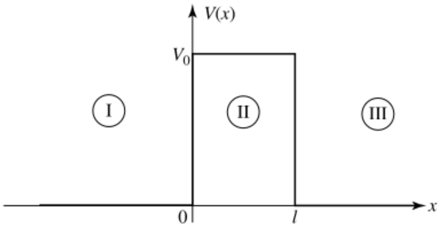
B) Caso $E<V_0$



$R=1$

Tarea.

Barreras de potencial



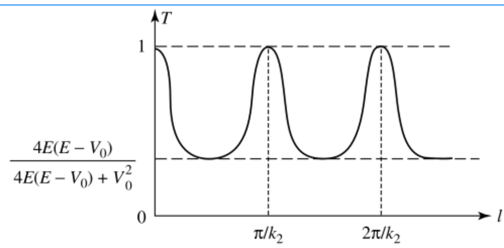
$$
\begin{aligned}
\varphi_{\mathrm{I}}(x) &= A_1\,\mathrm{e}^{ik_1x} + A_1'\,\mathrm{e}^{-ik_1x}\\
\varphi_{\mathrm{II}}(x) &= A_2\,\mathrm{e}^{ik_2x} + A_2'\,\mathrm{e}^{-ik_2x}\\
\varphi_{\mathrm{III}}(x) &= A_3\,\mathrm{e}^{ik_1x} + A_3'\,\mathrm{e}^{-ik_1x}
\end{aligned}
$$

$$R=\left|\frac{A_1'}{A_1}\right|^2=\frac{(k_1^2-k_2^2)^2\sin^2 k_2l}{4k_1^2k_2^2+(k_1^2-k_2^2)^2\sin^2 k_2l}$$

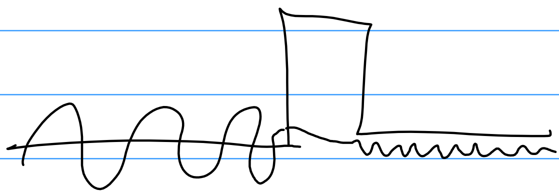
$$T=\left|\frac{A_3}{A_1}\right|^2=\frac{4k_1^2k_2^2}{4k_1^2k_2^2+(k_1^2-k_2^2)^2\sin^2 k_2l}$$

It is then easy to verify that $R+T=1$. Taking (9)

$$T=\frac{4E(E-V_0)}{4E(E-V_0)+V_0^2\sin^2\left[\sqrt{2m(E-V_0)}\,l/\hbar\right]}$$



Para $E<V_0$ esto es un modelo de jugete para efecto tunel



Cambiando

$k_2 \longrightarrow -i\rho_2$

$$T=\left|\frac{A_3}{A_1}\right|^2=\frac{4E(V_0-E)}{4E(V_0-E)+V_0^2\,\sinh^2\left[\sqrt{2m(V_0-E)}\,l/\hbar\right]}$$

$$\sinh x = \frac{e^{x}-e^{-x}}{2}$$

with, of course, $R=1-T$. When $\rho_2 l \gg 1$, we have:

$$T\simeq\frac{16E(V_0-E)}{V_0^2}\,\mathrm{e}^{-2\rho_2 l}$$