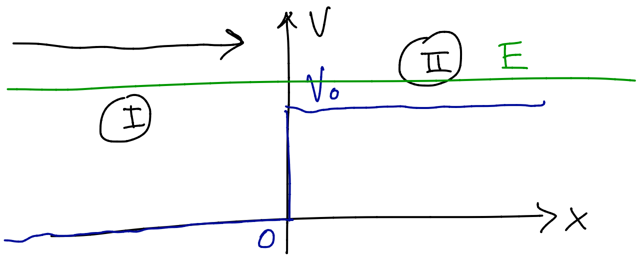
Capítulo III, BIII, sección 1 y 2. (Cohen) (Zettili)
Capítulo I, HI, sección 2a

Potenciales unidimensionales cuadrados

Procedimiento

1. Dividir en regiones de potencial cte.
2. Empalmar soluciones requiriendo la continuidad de $\varphi(x)$ y $\frac{d\varphi}{dx}$ en las discontinuidades del potencial.

a) Escalón de potencial



Caso $E > V_0$

(I) $\frac{d^2}{dx^2}\varphi_I + \underbrace{\frac{2m}{\hbar^2}E}_{\sqrt{k_1}}\varphi_I = 0$

(II) $\frac{d^2}{dx^2}\varphi_{II} + \underbrace{\frac{2m}{\hbar^2}(E - V_0)}_{\sqrt{k_2}}\varphi_{II} = 0$

$$\frac{d^2}{dx^2}\varphi_I + k_1^2\varphi_I = 0$$
$$\frac{d^2}{dx^2}\varphi_{II} + k_2^2\varphi_{II} = 0$$

$$\boxed{\begin{aligned}\varphi_I &= A_1 e^{ik_1x} + A_1' e^{-ik_1x}\\ \varphi_{II} &= A_2 e^{ik_2x} + \cancel{A_2' e^{-ik_2x}}\end{aligned}}$$

$A_2' = 0$

$\varphi_I(0) = \varphi_{II}(0)$ $\qquad$ $\varphi_I'(0) = \varphi_{II}'(0)$

$A_1 + A_1' = A_2 + A_2'$ $\qquad$ $ik_1A_1 - ik_1A_1' = ik_2A_2 - ik_2A_2'$

Recordemos la definición de corriente de probabilidad

$$\vec J(\vec r,t) = \frac{\hbar}{2mi}\left[\psi^*(\vec r,t)\nabla\psi(\vec r,t) - \underbrace{\psi(\vec r,t)\nabla\psi(\vec r,t)^*}_{c.c.}\right]$$

$$\varphi_I^*\nabla\varphi_I = \varphi_I^*\frac{d}{dx}\varphi_I = \left(A_1^* e^{-ik_1x} + A_1'^* e^{ik_1x}\right)\left(ik_1A_1e^{ik_1x} - ik_1A_1'e^{-ik_1x}\right)$$

$$= ik_1|A_1|^2 - \cancel{ik_1A_1^*A_1' e^{-2ik_1x}} + \cancel{ik_1A_1'^*A_1e^{2ik_1x}} - ik_1|A_1'|^2$$

$$-\quad \varphi_I\nabla\varphi_I^* = -ik_1|A_1|^2 + \cancel{ik_1A_1A_1'^*e^{2ik_1x}} - \cancel{ik_1A_1'A_1^*e^{-2ik_1x}} + ik_1|A_1'|^2$$

$$\varphi_I^*\nabla\varphi_I - \varphi_I\nabla\varphi_I^* = 2ik_1|A_1|^2 - 2ik_1|A_1'|^2 = 2ik_1\left(|A_1|^2 - |A_1'|^2\right)$$

$$\psi_I^* \nabla\psi_I - \psi_I \nabla\psi^* = 2ik_1\left(|A_1|^2 - |A_1'|^2\right)$$

$$J = \frac{\hbar}{2mi}\left(2ik_1\left(|A_1|^2 - |A_1'|^2\right)\right)$$

$$= \frac{\hbar}{m}k_1\left(|A_1|^2 - |A_1'|^2\right)$$

Basados en esto asociamos $\frac{\hbar k_1}{m}|A_1'|^2$ a la corriente reflejada y $\frac{\hbar k_1}{m}|A_1|^2$ a la incidente.

El coeficiente de reflexión de la barrera será

$$R = \left|\frac{A_1'}{A_1}\right|^2$$

Para ψ_{II} analogamente $J_2 = \frac{\hbar k_2}{m}|A_2|^2$ será la corriente transmitida.

El coeficiente de transmisión será

$$T = \frac{J_2}{J_i} = \frac{\frac{\hbar k_2}{m}|A_2|^2}{\frac{\hbar k_1}{m}|A_1|^2} = \frac{k_2}{k_1}\left|\frac{A_2}{A_1}\right|^2$$

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(1) $A_1 + A_1' = A_2$

(2) $ik_1 A_1 - ik_1 A_1' = ik_2 A_2$

Sustituimos (1) en (2) $\quad ik_1 A_1 - ik_1 A_1' = ik_2(A_1 + A_1')$

$$\Rightarrow (k_1 - k_2)A_1 = (k_2 + k_1)A_1' \Rightarrow \boxed{\frac{A_1'}{A_1} = \frac{k_1 - k_2}{k_1 + k_2}}$$

De (1) $A_1' = A_2 - A_1$ y sustituimos en (2)

$$k_1 A_1 - k_1(A_2 - A_1) = k_2 A_2$$

$$2k_1 A_1 = (k_1 + k_2)A_2 \Rightarrow \boxed{\frac{A_2}{A_1} = \frac{2k_1}{k_1 + k_2}}$$

$$\boxed{\frac{A_1'}{A_1} = \frac{k_1 - k_2}{k_1 + k_2}} \qquad \boxed{\frac{A_2}{A_1} = \frac{2k_1}{k_1+k_2}}$$

$$R = \left|\frac{A_1'}{A_1}\right|^2 = \frac{(k_1 - k_2)^2}{(k_1+k_2)^2} = \frac{k_1^2 - 2k_1k_2 + k_2^2}{(k_1+k_2)^2}$$

$$= \frac{(k_1+k_2)^2}{(k_1+k_2)^2} - \frac{4k_1k_2}{(k_1+k_2)^2}$$

$$\boxed{R = 1 - \frac{4k_1k_2}{(k_1+k_2)^2}} \qquad \boxed{T = \frac{4k_1k_2}{(k_1+k_2)^2}}$$

$$T = \frac{k_2}{k_1}\left|\frac{A_2}{A_1}\right|^2 = \frac{k_2}{k_1}\frac{4k_1^2}{(k_1+k_2)^2} = \frac{4k_1k_2}{(k_1+k_2)^2}$$

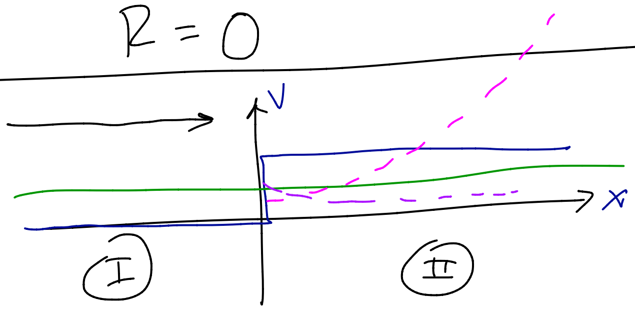
Notar que $R + T = 1$

$E >> V_0$ $\quad \sqrt{k_1} = \frac{2m}{\hbar^2}E \;;\; \sqrt{k_2} = \frac{2m}{\hbar^2}(E - V_0)$

$$\Rightarrow k_1 \simeq k_2 = k \quad \Rightarrow \quad T = \frac{4k^2}{(2k)^2} = 1$$

$$R = 0$$

$E < V_0$



(I) " lo mismo que $E > V_0$

(II) $\frac{d^2}{dx^2}\varphi_{II} - \underbrace{\frac{2m}{\hbar^2}(V_0 - E)}_{\sqrt{\beta_2}}\varphi_{II} = 0 \Rightarrow \boxed{\varphi_{II}'' - \beta_2^2\varphi_{II} = 0}$

$$\varphi_I = A_1 e^{ik_1x} + A_1' e^{-ik_1x}$$

$$\varphi_{II} = \underbrace{\cancel{B_2 e^{\beta_2 x}} + B_2' e^{-\beta_2 x}}_{B_2 = 0} = B_2' e^{-\beta_2 x}$$

$$\varphi_{II} = B_2 e^{\beta_2 x} + B_2' e^{-\beta_2 x}$$

$$\psi^* \frac{d}{dx}\psi = \left(B_2^* e^{\beta_2 x} + B_2'^* e^{-\beta_2 x}\right)\left(\beta_2 B_2 e^{\beta_2 x} - \beta_2 B_2' e^{-\beta_2 x}\right)$$

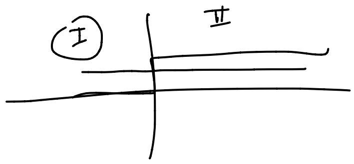
$$= \cancel{\beta_2 |B_2|^2 e^{2\beta_2 x}} - \beta_2 B_2^* B_2' + \beta_2 B_2'^* B_2 - \cancel{\beta_2 |B_2'|^2 e^{-2\beta_2 x}}$$

$$\psi \left(\frac{d}{dx}\psi\right)^* = \left(B_2 e^{\beta_2 x} + B_2' e^{-\beta_2 x}\right)\left(\beta_2 B_2^* e^{\beta_2 x} - \beta_2 B_2'^* e^{-\beta_2 x}\right)$$

$$= \cancel{\beta_2 |B_2|^2 e^{2\beta_2 x}} - \beta_2 B_2 B_2'^* + \beta_2 B_2' B_2^* - \cancel{\beta_2 |B_2'|^2 e^{-2\beta_2 x}}$$

$$\psi^* \psi' - \psi \psi^{*\prime} = 2\beta_2\left(B_2'^* B_2 - B_2^* B_2'\right)$$

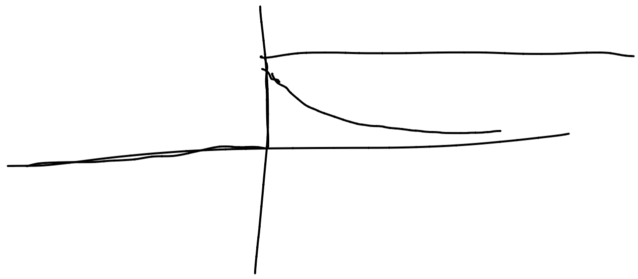
$$J = \frac{\beta_2}{2mi}\left(B_2'^* B_2 - B_2^* B_2'\right)$$



$$\varphi_I = A_1 e^{ik_1 x} + A_1' e^{-ik_1 x}$$

$$\varphi_{II} = B_2' e^{-\beta_2 x}$$

$$\frac{A_1'}{A_1} = \frac{k_1 - i\beta_2}{k_1 + i\beta_2}$$



$$R = \left|\frac{A_1'}{A_1}\right|^2 = 1$$