Suma de momentos angulares

Dos bases (del mismo espacio)

(desacoplada) misma dim. (acoplada)

$$|j_1 m_1 j_2 m_2\rangle \longleftrightarrow |j_1 j_2 j m\rangle$$

$J_1^2, J_{1z}, J_2^2, J_{2z}$ $\qquad$ $J_1^2\ J_2^2\ J^2\ J_z$

$$\vec{J}_1 + \vec{J}_2 = \vec{J}$$

Para pasar de una base a otra:

- Escribir J^2 y J_z en la base $|j_1 m_1 j_2 m_2\rangle$ y diagonalizar.

  - $J_z = J_{1z} + J_{2z}$
  - $J^2 = (\vec{J}_1 + \vec{J}_2)^2 = J_1^2 + J_2^2 + 2\vec{J}_1\cdot\vec{J}_2$

$$= J_1^2 + J_2^2 + 2J_{1z}J_{2z} + J_{1+}J_{2-} + J_{1-}J_{2+}$$

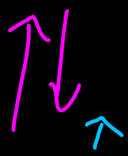
Obtenemos $|j_1 j_2 j m\rangle = \sum_{j_1' m_1' j_2' m_2'} C_{jm}^{j_1' m_1' j_2' m_2'} |j_1' m_1' j_2' m_2'\rangle$

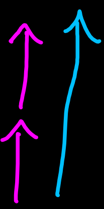
Coeficientes de Clebsch-Gordan $\langle jm|j_1 m_1 j_2 m_2\rangle$

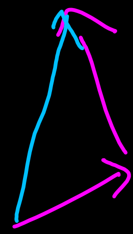
- $C_{jm}^{j_1 m_1 j_2 m_2}$ son cero a menos que:

  - $m_1 + m_2 = m$ $\quad (J_z = J_{1z} + J_{2z})$

• $|j_1 - j_2| \le j \le j_1 + j_2$







Ojo $j \ne j_1 + j_2$

— Ejemplo: $j_1 = 1/2 \qquad j_2 = 1/2$

$|j_1 j_2 j m\rangle = |j m\rangle \qquad |j_1 m_1 j_2 m_2\rangle = |\epsilon_1 \epsilon_2\rangle$

$\rightarrow |0 0\rangle = (|+-\rangle - |-+\rangle)/\sqrt{2}$

$|1 1\rangle = |++\rangle$

$|1 0\rangle = (|+-\rangle + |-+\rangle)/\sqrt{2}$

$|1 -1\rangle = |--\rangle$

Ejemplo 2: $j_1 = \frac{1}{2} \qquad j_2 = 1$

Base desacoplada

$|j_1 m_1 j_2 m_2\rangle \rightarrow |\tfrac{1}{2} -\tfrac{1}{2}\ 1 -1\rangle, |\tfrac{1}{2}\ \tfrac{1}{2}\ 1 -1\rangle$

Usando $\quad |\tfrac{1}{2} -\tfrac{1}{2}\ 1\ 0\rangle, |\tfrac{1}{2}\ \tfrac{1}{2}\ 1\ 0\rangle$

$-j_1 < m_1 < j_1 \quad |\tfrac{1}{2} -\tfrac{1}{2}\ 1 +1\rangle, |\tfrac{1}{2}\ \tfrac{1}{2}\ 1 +1\rangle$

$-j_2 < m_2 < j_2$

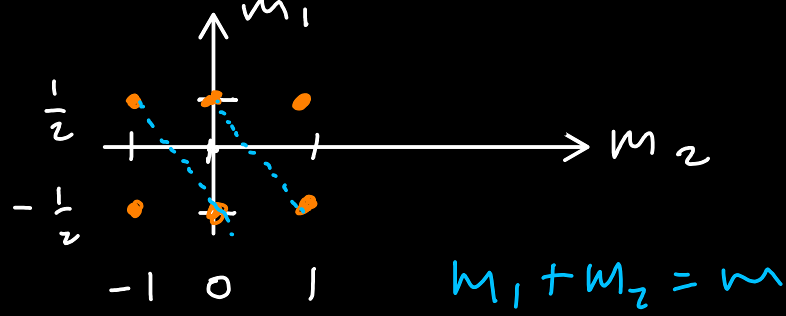
Base acoplada

$|j m\rangle \rightarrow |\tfrac{1}{2} -\tfrac{1}{2}\rangle \quad |\tfrac{1}{2}\ \tfrac{1}{2}\rangle$

$\frac{1}{2} = |j_1 - j_2| \le j \le j_1 + j_2 = 3/2 \quad |\tfrac{3}{2} -\tfrac{3}{2}\rangle, |\tfrac{3}{2} -\tfrac{1}{2}\rangle, |\tfrac{3}{2}\ \tfrac{1}{2}\rangle, |\tfrac{3}{2}\ \tfrac{3}{2}\rangle$

$-j < m < j$

¿Cómo relacionar ambas bases?



Empezando con un estado extremo

$$\underbrace{\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle}_{|j\ m\rangle} = \sum_{\substack{m_1+m_2=m \\ |j_1-j_2|\le j\le j_1+j_2}} C_{jm}^{j_1m_1\,j_2m_2}\,|j_1 m_1 j_2 m_2\rangle = \left|\tfrac{1}{2}\ \tfrac{1}{2}\ 1\ 1\right\rangle$$

$$\left|\tfrac{3}{2}\ -\tfrac{3}{2}\right\rangle = \left|\tfrac{1}{2}\ -\tfrac{1}{2}\ 1\ -1\right\rangle$$

$$\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle \underset{J_+}{\overset{J_-}{\rightleftarrows}} \left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle \underset{J_+}{\overset{J_-}{\rightleftarrows}} \left|\tfrac{3}{2}\ -\tfrac{1}{2}\right\rangle \underset{J_+}{\overset{J_-}{\rightleftarrows}} \left|\tfrac{3}{2}\ -\tfrac{3}{2}\right\rangle$$

$$J_- = J_{1-} + J_{2-}$$

$$J_-\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle = (J_{1-}+J_{2-})\left|\tfrac{1}{2}\ \tfrac{1}{2}\ 1\ 1\right\rangle$$

$$\hbar\sqrt{A}\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle = J_{1-}\left|\tfrac{1}{2}\ \tfrac{1}{2}\ 1\ 1\right\rangle + J_{2-}\left|\tfrac{1}{2}\ \tfrac{1}{2}\ 1\ 1\right\rangle$$

$$= \hbar\sqrt{B}\left|\tfrac{1}{2}\ -\tfrac{1}{2}\ 1\ 1\right\rangle + \hbar\sqrt{C}\left|\tfrac{1}{2}\ \tfrac{1}{2}\ 1\ 0\right\rangle$$

$$\therefore\ \left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle = \frac{\sqrt{B}\left|\tfrac{1}{2}\ -\tfrac{1}{2}\ 1\ 1\right\rangle + \sqrt{C}\left|\tfrac{1}{2}\ \tfrac{1}{2}\ 1\ 0\right\rangle}{\sqrt{A}}$$

Para obtener los que nos faltan

$$\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle = \alpha\left|\tfrac{1}{2}\ \tfrac{1}{2}\ 1\ 0\right\rangle + \beta\left|\tfrac{1}{2}\ -\tfrac{1}{2}\ 1\ 1\right\rangle$$

usando $m = m_1 + m_2$

Obtenemos α y β notando que:

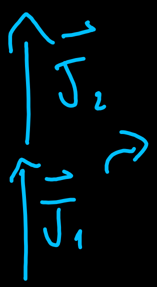
* $\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$ debe estar normalizado
* $\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$ debe ser ortogonal a $\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle$

Esto define α y β salvo por una fase global que podemos elegir

Luego $\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle \xrightarrow{J_-} \left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle$ $\blacksquare$

En general:

Primer nivel: $j = j_1 + j_2$



$$|j = j_1 + j_2,\ m = j_1 + j_2\rangle = |j_1, m_1 = j_1,\ j_2, m_2 = j_2\rangle$$

- Luego obtenemos

$$|j = j_1 + j_2,\ m = j_1 + j_2 - 1\rangle,\ |j = j_1 + j_2,\ m = j_1 + j_2 - 2\rangle, \dots$$

usando $J_- = J_{1-} + J_{2-}$

Siguiente nivel: $j = j_1 + j_2 - 1$

$$|j = j_1 + j_2 - 1,\ m = j_1 + j_2 - 1\rangle = \alpha\,|j_1\, m_1 = j_1 - 1,\ j_2, m_2 = j_2\rangle + \beta\,|j_1\, m_1 = j_1,\ j_2, m_2 = j_2 - 1\rangle$$

Determinamos α y β con: * normalización

* ortogonalidad con $|j = j_1 + j_2,\ m = j_1 + j_2 - 1\rangle$

Usamos J_- para obtener los demás con $j = j_1 + j_2 - 1$

Siguiente nivel: $j = j_1 + j_2 - 2$ (si $|j_1 - j_2| \leq j_1 + j_2 - 2$)

$$|j = j_1 + j_2 - 2,\ m = j_1 + j_2 - 2\rangle = \left.\begin{array}{l} \alpha\,| \ -2 \qquad\qquad \rangle + \\ \beta\,| \qquad\qquad -2\ \rangle + \\ \gamma\,| \ -1 \qquad -1\ \rangle \end{array}\right\}$$

obtenemos α, β, γ con:
* normalización
* ortogonalidad $m = j_1 + j_2 - 2$

Usamos J_- para obtener los demás con $j = j_1 + j_2 - 2$.

Suponiendo $j = j_1 + j_2$ es entero

$j \downarrow \quad m \rightarrow \quad \ldots \; -1 \quad 0 \quad +1 \ldots$

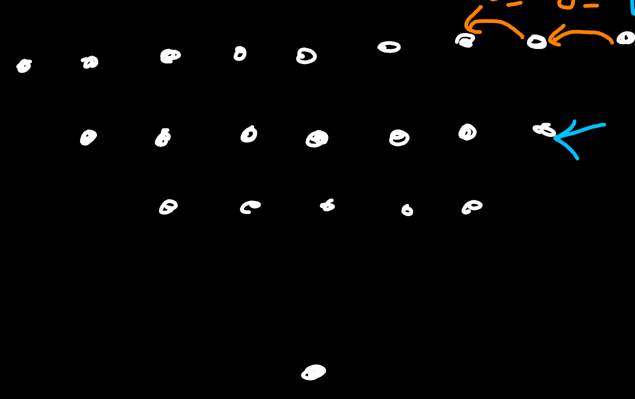
$j_1 + j_2$

$j_1 + j_2 - 1$

$j_1 + j_2 - 2$

$\vdots$

$|j_1 - j_2|$

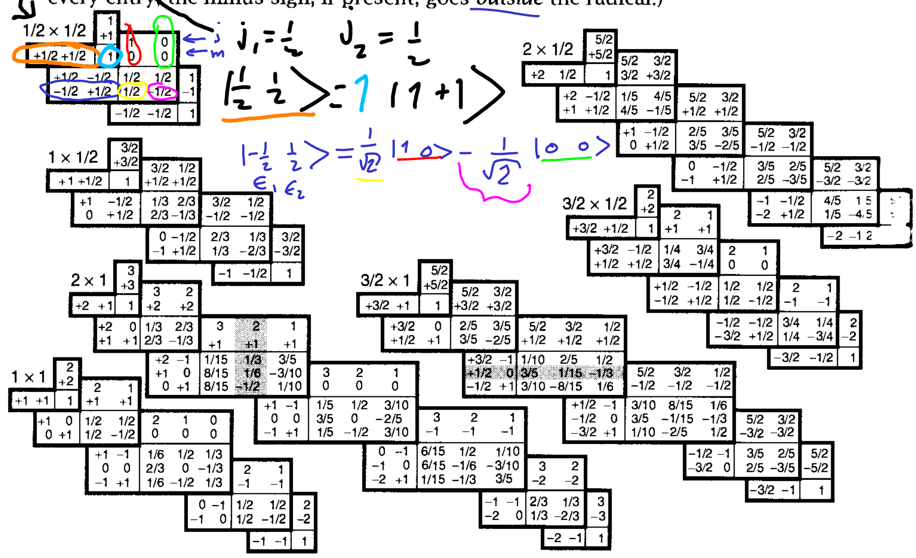


$J_- \quad J_-$

$|J = j_1 + j_2,\; m = j_1 + j_2\rangle$
$\|$
$|j_1\, m_1 = j_1,\; j_2\; m_2 = j_2\rangle$

En la practica

Table 4.7: Clebsch-Gordan coefficients. (A square root sign is understood for every entry; the minus sign, if present, goes *outside* the radical.)



$j_1 = \frac{1}{2} \quad j_2 = \frac{1}{2}$

$|\frac{1}{2}\;\frac{1}{2}\rangle = 1\;|1\;+1\rangle$

$|-\frac{1}{2}\;\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\,|1\;0\rangle - \frac{1}{\sqrt{2}}\,|0\;0\rangle$

$\epsilon_1\, \epsilon_2$

ClebschGordan

ClebschGordan[{j_1, m_1}, {j_2, m_2}, {j, m}]
gives the Clebsch–Gordan coefficient for the decomposition of $|j, m\rangle$ in terms of $|j_1, m_1\rangle |j_2, m_2\rangle$.

Run code block in SymPy Live

```
>>> from sympy.physics.quantum.cg import CG
>>> from sympy import S
>>> cg = CG(S(3)/2, S(3)/2, S(1)/2, -S(1)/2, 1, 1)
>>> cg
CG(3/2, 3/2, 1/2, -1/2, 1, 1)
>>> cg.doit()
sqrt(3)/2
```


Table of Clebsch–Gordan coefficients

From Wikipedia, the free encyclopedia

This is a **table of Clebsch–Gordan coefficients** used for adding angular momentum values in quantum mechanics. The overall sign of the coefficients for each set of constant j_1, j_2, j is arbitrary to some degree and has been fixed according to the Condon–Shortley and Wigner sign convention as discussed by Baird and Biedenharn.[1] Tables with the same sign convention may be found in the Particle Data Group's *Review of Particle Properties*[2] and in online tables.[3]

Contents [show]

Formulation [edit]

The Clebsch–Gordan coefficients are the solutions to

$$|j_1, j_2; J, M\rangle = \sum_{m_1=-j_1}^{j_1} \sum_{m_2=-j_2}^{j_2} |j_1, m_1; j_2, m_2\rangle \langle j_1, j_2; m_1, m_2 \mid j_1, j_2; J, M\rangle$$

Explicitly:

$$\langle j_1, j_2; m_1, m_2 \mid j_1, j_2; J, M\rangle$$

$$= \delta_{M, m_1+m_2} \sqrt{\frac{(2J+1)(J+j_1-j_2)!(J-j_1+j_2)!(j_1+j_2-J)!}{(j_1+j_2+J+1)!}} \times$$

$$\sqrt{(J+M)!(J-M)!(j_1-m_1)!(j_1+m_1)!(j_2-m_2)!(j_2+m_2)!} \times$$

Otra notación

Símbolos 3j de Wigner

$$\begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & -m \end{pmatrix} \sqrt{2j+1}\,(-1)^{m+j_1-j_2} = \langle j_1 m_1 j_2 m_2 \mid j\, m\rangle$$