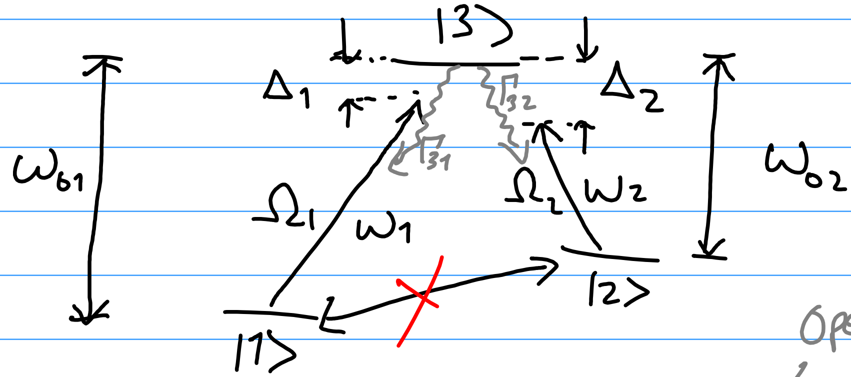
Átomo de 3 niveles con más detalle



Operador de Lindblad
$\mathcal{L}[A]\rho = A\rho A^{+} - \frac{1}{2}\left(A^{+}A\rho + \rho A A^{+}\right)$

$$\dot{\rho}_A = \frac{1}{i\hbar}\left[\tilde{H}, \rho_A\right] + \Gamma_{31}\mathcal{L}[\sigma_{13}]\rho_A$$

$$+ \Gamma_{32}\mathcal{L}[\sigma_{23}]\rho_A$$ ← clave pasada decía 32

$$+ \gamma_{2dec}\mathcal{L}[\sigma_{22}]\rho_A$$

$$+ \gamma_{1dec}\mathcal{L}[\sigma_{11}]\rho_A$$ ← no se pone porque el esfasamiento es relativo

$$+ \gamma_{3dec}\mathcal{L}[\sigma_{33}]\rho_A$$

evolución coherente

Ecuación maestra.

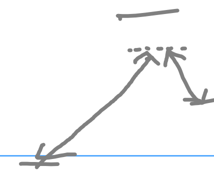
decoherencia
(evolución no unitaria)

$$\tilde{H} = \hbar\Delta_1|1\rangle\langle 1| + \hbar\Delta_2|2\rangle\langle 2| + \frac{\hbar}{2}\Omega_1\left(\sigma_{13}e^{-i\vec{k}_1\cdot\vec{r}} + \sigma_{13}^{+}e^{i\vec{k}_1\cdot\vec{r}}\right)$$

$$+ \frac{\hbar}{2}\Omega_2\left(\sigma_{23}e^{-i\vec{k}_2\cdot\vec{r}} + \sigma_{23}^{+}e^{i\vec{k}_2\cdot\vec{r}}\right)$$

$$\tilde{\Omega}_j = \Omega_j e^{i\vec{k}_j\cdot\vec{r}}$$

- Eliminación adiabática.



Ocurre cuando $\Delta_j >> \Gamma_{31}, \Gamma_{32}, \Omega_1, \Omega_2$
con $|\Delta_1 - \Delta_2| < \Omega_1, \Omega_2, \Gamma_{31}, \Gamma_{32}$

$$i\hbar \partial_t |\psi\rangle = \tilde{H} |\psi\rangle \quad ; \quad |\psi\rangle = \alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle$$

Usando la base en el orden $|1\rangle, |2\rangle, |3\rangle$

$$i\cancel{\hbar}\partial_t \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \cancel{\hbar} \begin{pmatrix} \Delta_1 & 0 & \tilde{\Omega}_1^*/2 \\ 0 & \Delta_2 & \tilde{\Omega}_2^*/2 \\ \tilde{\Omega}_1/2 & \tilde{\Omega}_2/2 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$\begin{aligned} i\dot{\alpha} &= \Delta_1 \alpha + \tilde{\Omega}_1^*/2\, \gamma \\ i\dot{\beta} &= \Delta_2 \beta + \tilde{\Omega}_2^*/2\, \gamma \\ i\dot{\gamma} &= \tilde{\Omega}_1/2\, \alpha + \tilde{\Omega}_2/2\, \beta \end{aligned}$$

Haciendo una transformación unitaria

$$U_2 = \begin{pmatrix} e^{i\Delta t} & 0 & 0 \\ 0 & e^{i\Delta t} & 0 \\ 0 & 0 & e^{i\Delta t} \end{pmatrix} \quad \text{con} \quad \Delta = \frac{1}{2}(\Delta_1 + \Delta_2)$$

$$\tilde{\tilde{H}} = U^\dagger \tilde{H} U + i\hbar (\partial_t U^\dagger) U$$

Cambiando el cero de energía

$$i\cancel{\hbar}\partial_t \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \cancel{\hbar} \begin{pmatrix} \Delta_1 - \Delta & 0 & \tilde{\Omega}_1^*/2 \\ 0 & \Delta_2 - \Delta & \tilde{\Omega}_2^*/2 \\ \tilde{\Omega}_1/2 & \tilde{\Omega}_2/2 & -\Delta \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$i\dot{\alpha} = (\Delta_1 - \Delta)\,\alpha + \frac{\tilde{\Omega}_1^*}{2}\,\gamma$$
$$i\dot{\beta} = (\Delta_2 - \Delta)\,\beta + \tilde{\Omega}_2^*/2\;\gamma$$
$$i\dot{\gamma} = \tilde{\Omega}_1/2\;\alpha + \tilde{\Omega}_2/2\;\beta - \Delta\gamma$$

Método formal de eliminación adiabática en artículo de Mølmer "Adiabatic elimination"

$$\dot{\gamma} = 0 \Rightarrow \gamma = \frac{1}{2}\left(\tilde{\Omega}_1\alpha + \tilde{\Omega}_2\beta\right)/\Delta$$

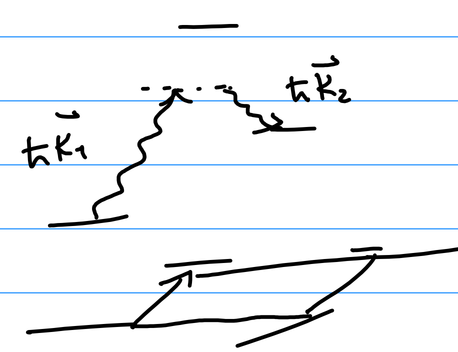
$$i\dot{\alpha} = (\Delta_1 - \Delta)\,\alpha + \frac{\tilde{\Omega}_1^*}{4\Delta}\left(\tilde{\Omega}_1\alpha + \tilde{\Omega}_2\beta\right)$$

$$i\dot{\beta} = (\Delta_2 - \Delta)\,\beta + \frac{\tilde{\Omega}_2^*}{4\Delta}\left(\tilde{\Omega}_1\alpha + \tilde{\Omega}_2\beta\right)$$

$$i\partial_t\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\Delta_1 - \Delta + \frac{|\tilde{\Omega}_1|^2}{4\Delta} & \frac{\tilde{\Omega}_1^*\tilde{\Omega}_2}{4\Delta}\\ \frac{\tilde{\Omega}_1\tilde{\Omega}_2^*}{4\Delta} & \Delta_2 - \Delta + \frac{|\tilde{\Omega}_2|^2}{4\Delta}\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\omega_{eff_1} & \frac{\Omega_{eff}}{2}\\ \frac{\Omega_{eff}}{2} & \omega_{eff_2}\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

$$\Omega_{eff} = \frac{\tilde{\Omega}_1\tilde{\Omega}_2^*}{2\Delta} = \frac{\Omega_1\Omega_2}{2\Delta}\,e^{-i(\vec{k}_2 - \vec{k}_1)\cdot\vec{r}}$$

$$e^{-i\vec{k}\cdot\vec{r}}\,|\vec{p}\rangle = |\vec{p} - \hbar\vec{k}\rangle$$



$$\omega_{eff_j} = \Delta_j - \Delta + \frac{|\tilde{\Omega}_j|^2}{4\Delta}$$

$$H_{eff} = \hbar\omega_{eff_1}|1\rangle\langle 1| + \hbar\omega_{eff_2}|2\rangle\langle 2| + \frac{\hbar}{2}\left(\Omega_{eff}^*\sigma_{12} + \Omega_{eff}\sigma_{12}^\dagger\right)$$

$\Delta = \frac{1}{2}(\Delta_1 + \Delta_2)$

La resonancia no ocurre con $\Delta_1 = \Delta_2$.
Entonces ¿Cuándo?

$$\Delta_1 - \Delta + \frac{|\Omega_1|^2}{4\Delta} = \Delta_2 - \Delta + \frac{|\Omega_2|^2}{4\Delta}$$

$$\frac{\Delta_1}{2} - \frac{\Delta_2}{2} + \frac{|\Omega_1|^2}{4\Delta} = \frac{\Delta_2}{2} - \frac{\Delta_1}{2} + \frac{|\Omega_2|^2}{4\Delta}$$

$$\Delta_1 - \Delta_2 = \underbrace{\frac{|\Omega_2|^2}{4\Delta} - \frac{|\Omega_1|^2}{4\Delta}}_{\text{light Shift}}$$

ac-Stark shift
e.v. de estados vestidos