Átomo de dos niveles interactuando con un campo periódico

$|g\rangle, \; |e\rangle \qquad ; \qquad |\uparrow\rangle, \; |\downarrow\rangle$

$H = \hbar\omega_0 |e\rangle\langle e| - \vec{d}\cdot\vec{\mathcal{E}}(t) \quad ; \quad H = \hbar\omega_0 |\uparrow\rangle\langle\uparrow| - \vec{\mu}_s \cdot \vec{B}(t)$

aproximación dipolar

$$\vec{\mathcal{E}}(t) = \hat{\epsilon}\, \mathcal{E}_0 \cos(\omega t) = \hat{\epsilon}\frac{\mathcal{E}_0}{2}\left(e^{-i\omega t} + e^{i\omega t}\right) = \mathcal{E}^{(+)}(t) + \mathcal{E}^{(-)}(t)$$

Vamo a suponer que $|e\rangle$ y $|g\rangle$ son de paridad defindia y opuesta, y además

$$\underbrace{\langle g|\vec{d}|e\rangle}_{\in \mathbb{R}} \neq 0. \quad \text{y} \quad \langle g|\vec{d}|g\rangle = 0, \quad \langle e|\vec{d}|e\rangle = 0$$

$$\vec{d} = \mathbb{1}\,\vec{d}\,\mathbb{1} = (|e\rangle\langle e| + |g\rangle\langle g|)\,\vec{d}\,(|e\rangle\langle e| + |g\rangle\langle g|)$$

$|e\rangle\langle e| + |g\rangle\langle g|$

$$= |e\rangle\langle e|\vec{d}|g\rangle\langle g| + |g\rangle\langle g|\vec{d}|e\rangle\langle e|$$

$$= \langle e|\vec{d}|g\rangle \left(|e\rangle\langle g| + |g\rangle\langle e|\right)$$

$$= \langle e|\vec{d}|g\rangle \left(\sigma^{+} + \sigma\right)$$

$\sigma = |g\rangle\langle e|$

$$= d^{(+)} + d^{(-)}$$

$$H_{AF} = -\vec{d}\cdot\vec{\mathcal{E}} = -\overset{-\omega+\omega_0}{\overbrace{d^{(+)}\cdot\mathcal{E}^{(-)}}} - \overset{\omega-\omega_0}{\overbrace{d^{(-)}\cdot\mathcal{E}^{(+)}}} - \overset{-\omega-\omega_0}{\overbrace{d^{(-)}\cdot\mathcal{E}^{(-)}}} - \overset{\omega+\omega_0}{\overbrace{d^{(+)}\mathcal{E}^{(+)}}}$$

$$H_A = \hbar\omega_0 |e\rangle\langle e| \Rightarrow U(t) = e^{-i\omega_0 |e\rangle\langle e| t}$$

$$\langle\sigma(t)\rangle \sim e^{-i\omega_0 t} \qquad \langle\sigma^\dagger(t)\rangle \sim e^{i\omega_0 t}$$

Para Rb en edo base interactuando con luz cerca de resonancia (comparado con el ancho de la transición).

$$\frac{\omega_0}{2\pi} \sim 380\ \text{THz} \qquad \Gamma = 6\ \text{MHz}$$

$$\frac{\omega}{2\pi} \sim 380\ \text{THz}$$

Si $\delta = \omega - \omega_0 \sim \Gamma$ podemos hacer la aproximación de onda rotante.

Para el Laboratorio de Materia Ultrafría

$$\underset{280\ \text{THz}}{\lambda_{ODT} = 1070\ \text{nm}} \qquad \underset{447\ \text{THz}}{\lambda_{D_2} = 670\ \text{nm}}$$

$$\delta \sim 2\pi \times 167\ \text{THz}$$

$\omega - \omega_0 \sim \omega + \omega_0$ y la RWA no es tan correcta.

Usando la aproximación de onda rotante (RWA)

$$H_{AF} = -\langle g|\hat{\epsilon}\cdot\vec{d}|e\rangle \left(\mathcal{E}_0^{(-)} \sigma e^{i\omega t} + \mathcal{E}_0^{(+)} \sigma^{+} e^{-i\omega t} \right)$$

$$= \frac{\hbar\Omega}{2}\left(\sigma e^{i\omega t} + \sigma^{+} e^{-i\omega t} \right)$$

$$\Omega = -\frac{\langle g|\hat{\epsilon}\cdot\vec{d}|e\rangle \mathcal{E}_0}{\hbar}$$ (frecuencia de Rabi)

Marco rotante $U = \exp(i\omega|e\rangle\langle e|)$

$$|\tilde{\psi}\rangle = U|\psi\rangle$$

$$\tilde{H} = UHU + i\hbar(\partial_t U)U^{+}$$ (Ver Steck o notas de FAMC)

$$\tilde{H} = \underbrace{-\hbar\,\delta|e\rangle\langle e|}_{\tilde{H}_A} + \underbrace{\frac{\hbar\Omega}{2}(\sigma+\sigma^{+})}_{\tilde{H}_{AF}}$$

$$\tilde{H} = \hbar\begin{pmatrix} 0 & \Omega/2 \\ \Omega/2 & -\delta \end{pmatrix}$$

$$\Delta E^{(2)} = \sum \frac{|\langle n|H'|m\rangle|^2}{E}$$

$$\Delta E^{(1)}_{|g\rangle} = 0 \qquad \Delta E^{(1)}_{|e\rangle} = 0$$

$$\Delta E^{(2)}_{|g\rangle} = \frac{|\langle g|\tilde{H}_{AF}|e\rangle|^2}{E_g - E_e} = -\frac{\hbar^{\cancel{2}}\Omega^2}{2\cancel{\hbar}\omega_0} = -\frac{\hbar\Omega^2}{2\omega_0}$$

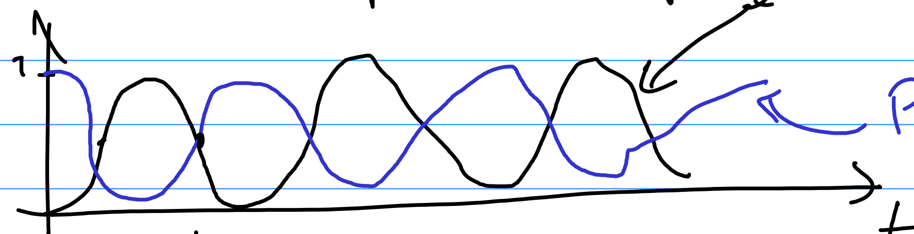
$$\Delta E^{(2)}_{|e\rangle} = \frac{\hbar}{2}\frac{\Omega^2}{\omega_0}$$

eigenvalores y eigenvectores

$$E_{\pm} = -\frac{\hbar\delta}{2} \pm \frac{\hbar}{2}\sqrt{\Omega^2+\delta^2}$$

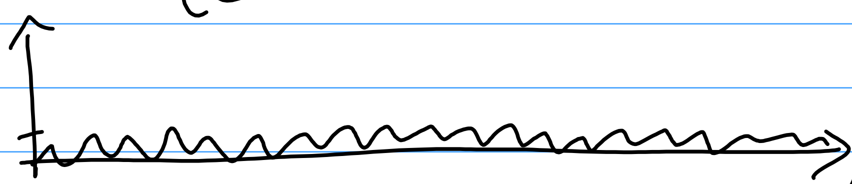
$$|+\rangle = \sin\theta\,|g\rangle + \cos\theta\,|e\rangle$$
$$|-\rangle = \cos\theta\,|g\rangle - \sin\theta\,|e\rangle \qquad \tan 2\theta = -\frac{\Omega}{\delta}$$

Dinámica temporal P_e en resonancia



P_g en resonancia

Fuera de resoncia



$$\tilde{\Omega} = \sqrt{\Omega^2+\delta^2}$$

— Para incluir decaimiento lo escribimos en términos de matriz densidad

$$\dot{\tilde{\rho}} = -\frac{i}{\hbar}[\tilde{H}, \tilde{\rho}]$$

$$\dot{\rho}_{ee} = i\Omega/2\,(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}) - \Gamma\rho_{ee}$$
$$\dot{\tilde{\rho}}_{ge} = -i\Omega/2\,(\rho_{ee} - \rho_{gg}) + i\delta\tilde{\rho}_{ge} - \frac{\Gamma}{2}\tilde{\rho}_{ge}$$
$$\dot{\tilde{\rho}}_{eg} = i\Omega/2\,(\rho_{ee} - \rho_{gg}) - i\delta\tilde{\rho}_{ge} - \Gamma/2\,\tilde{\rho}_{eg}$$
$$\dot{\rho}_{gg} = -i\Omega/2\,(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}) + \Gamma\rho_{ee}$$

Soluciones estacionarias

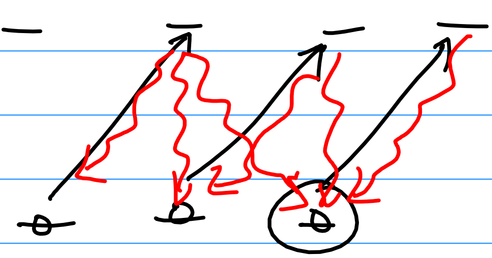
$$\rho_{ee}(t \to \infty) = \frac{\Omega^2/\Gamma^2}{1 + \left(\frac{2\delta}{\Gamma}\right)^2 + 2\left(\frac{\Omega}{\Gamma}\right)^2}$$

$$\tilde{\rho}_{eg}(t \to \infty) = -\frac{i\Omega}{\Gamma} \, \frac{1 + \frac{2i\delta}{\Gamma}}{1 + \left(\frac{2\delta}{\Gamma}\right)^2 + 2\frac{\Omega^2}{\Gamma^2}}$$

Para un átomo realista:

$$\begin{aligned}
\frac{\partial}{\partial t}\tilde{\rho}_{\alpha\, m_\alpha,\, \beta\, m_\beta} = -\frac{i}{2}\Bigg[& \delta_{\alpha \mathrm{e}} \sum_{m_\mathrm{g}} \Omega(m_\mathrm{g}, m_\alpha)\, \tilde{\rho}_{\mathrm{g}\, m_\mathrm{g},\, \beta\, m_\beta} - \delta_{\mathrm{g}\beta} \sum_{m_\mathrm{e}} \Omega(m_\beta, m_\mathrm{e})\, \tilde{\rho}_{\alpha\, m_\alpha,\, \mathrm{e}\, m_\mathrm{e}} \\
& + \delta_{\alpha \mathrm{g}} \sum_{m_\mathrm{e}} \Omega^*(m_\alpha, m_\mathrm{e})\, \tilde{\rho}_{\mathrm{e}\, m_\mathrm{e},\, \beta\, m_\beta} - \delta_{\mathrm{e}\beta} \sum_{m_\mathrm{g}} \Omega^*(m_\mathrm{g}, m_\beta)\, \tilde{\rho}_{\alpha\, m_\alpha,\, \mathrm{g}\, m_\mathrm{g}} \Bigg] \quad \text{(pump field)} \\
& - \delta_{\alpha\mathrm{e}}\delta_{\mathrm{e}\beta}\, \Gamma\, \tilde{\rho}_{\alpha\, m_\alpha,\, \beta\, m_\beta} \\
& - \delta_{\alpha\mathrm{e}}\delta_{\mathrm{g}\beta}\, \frac{\Gamma}{2}\, \tilde{\rho}_{\alpha\, m_\alpha,\, \beta\, m_\beta} \\
& - \delta_{\alpha\mathrm{g}}\delta_{\mathrm{e}\beta}\, \frac{\Gamma}{2}\, \tilde{\rho}_{\alpha\, m_\alpha,\, \beta\, m_\beta} \\
& + \delta_{\alpha\mathrm{g}}\delta_{\mathrm{g}\beta}\, \Gamma \sum_{q=-1}^{1} \Big[\tilde{\rho}_{\mathrm{e}\, (m_\alpha + q),\, \mathrm{e}\, (m_\beta + q)} \\
& \qquad \langle J_\mathrm{e}\, (m_\alpha + q) | J_\mathrm{g}\, m_\alpha; 1\, q \rangle \langle J_\mathrm{e}\, (m_\beta + q) | J_\mathrm{g}\, m_\beta; 1\, q \rangle \Big] \quad \text{(dissipation)} \\
& + \; i(\delta_{\alpha\mathrm{e}}\delta_{\mathrm{g}\beta} - \delta_{\alpha\mathrm{g}}\delta_{\mathrm{e}\beta})\, \Delta\, \tilde{\rho}_{\alpha\, m_\alpha,\, \beta\, m_\beta}. \quad \text{(free evolution)}
\end{aligned}$$

(master equation, fine-structure transition) (7.515)



$|e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\langle e| = (1\ 0)$

Esfera de Bloch

$\sigma = |g\rangle\langle e| = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma + \sigma^+$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = i(\sigma - \sigma^+)$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \sigma^+\sigma - \sigma\sigma^+ = [\sigma^+, \sigma]$$

$$\partial_t \langle \sigma_x \rangle =$$

$$\partial_t \langle \sigma_y \rangle =$$

$$\partial_t \langle \sigma_z \rangle =$$

$$\partial_t \overrightarrow{\langle \sigma \rangle} = \vec{P} \times \overrightarrow{\langle \sigma \rangle}$$

$$\vec{P} = \Omega \hat{x} - \Delta \hat{z}$$